Dodecagons for modal opposition in the Quantified Argument Calculus

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Abstract. We analyse dodecagons of opposition for *de re* and *de dicto* modalities in Quarc. The logical theories of the two dodecagons are encoded via inference trees; moreover, we provide a decidability result and a model-theoretic semantics for these theories.

Keywords: Modal Dodecagons · Inference Trees · Quarc.

Overview. The Quantified Argument Calculus (Quarc) is a logical system tailored to the syntax of natural languages [1]. In Quarc a quantifier forms, together with a unary predicate, an argument of predication; e.g., "every musician plays an instrument" is $(\forall M, \exists I)P$. As in [2], we extend the basic Quarc language with operators for necessity ($\Box$) and possibility ($\Diamond$), used either as sentential operators (e.g., "it is possible that Pegasus flies" is $\Diamond pF$) or as modes of predication (e.g., "Pegasus possibly flies" is $p\Diamond F$). We define twelve fundamental *de re* and *de dicto* modalities, graphically represent their logical relations via *dodecagons of opposition* and encode the resulting logical theories via lists of *inference trees*. Finally, we provide a model-theoretic semantics for the theories.

Formal language. Primitive symbols in our Quarc language $\mathcal{L}$ are: a set of unary predicates $\mathtt{Pred}$; operators for negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$), necessity ($\Box$) and possibility ($\Diamond$); universal quantifier ($\forall$) and particular quantifier ($\exists$); round brackets. The set of *basic modalities* is $\mathsf{MOD} = \{\epsilon, \neg, \Diamond, \Box, \Diamond\neg, \Box\neg, \neg\Diamond, \neg\Box, \neg\Diamond\neg, \neg\Box\neg\}$, where ϵ is an empty sequence. A basic modality is *proper* iff it includes $\Diamond$ or $\Box$. The following are pairs of *analogous modalities*: $\{\Diamond, \neg\Box\neg\}$, $\{\Box, \neg\Diamond\neg\}$, $\{\neg\Diamond, \Box\neg\}$ and $\{\neg\Box, \Diamond\neg\}$.

Grammar. The set of wffs in $\mathcal{L}$ is the smallest closed under the following clauses, where $S, R, P \in \mathtt{Pred}$, $\Pi, \Pi' \in \{\forall, \exists\}$, $\mathsf{m}_1, \mathsf{m}_2, \mathsf{m}_3, \mathsf{m}_4 \in \mathsf{MOD}$ and $\otimes \in \{\wedge, \vee\}$:

- $\mathsf{m}_1(\Pi S)\mathsf{m}_2 P$ is a wff if at most one between m_1 and m_2 is a proper modality;
- $\mathsf{m}_1(\Pi S)\mathsf{m}_2 P \otimes \mathsf{m}_3(\Pi' R)\mathsf{m}_4 P$ is a wff if at most one between m_1 and m_2, as well as between m_3 and m_4, is a proper modality.

We use ϕ, ψ, χ... for wffs, Γ, Δ, Θ... for sets of wffs and sometimes omit brackets.

De re modalities. Below are twelve *de re* modalities in $\mathcal{L}$. The label for a modality consists of two letters, the *first* being **U** (universality) or **P** (particularity), the *second* being **N** (necessity), **P** (possibility), **I** (impossibility), **V** (avoidability), **B** (absoluteness) or **C** (contintency). **UN**: $(\forall S)\Box P$; **UP**: $(\forall S)\Diamond P$; **PN**: $(\exists S)\Box P$; **PP**: $(\exists S)\Diamond P$; **UI**: $(\forall S)\Box\neg P$; **UV**: $(\forall S)\Diamond\neg P$; **PI**: $(\exists S)\Box\neg P$; **PV** $(\exists S)\Diamond\neg P$; **UB**: $(\forall S)\Box P \vee (\forall S)\Box\neg P$; **UC**: $(\forall S)\Diamond P \wedge (\forall S)\Diamond\neg P$; **PB**: $(\exists S)\Box P \vee (\exists S)\Box\neg P$; **PC**: $(\exists S)\Diamond P \wedge (\exists S)\Diamond\neg P$.

De dicto modalities. The following are twelve *de dicto* modalities in $\mathcal{L}$. Inverse labelling conventions apply. **NU**: $\Box(\forall S)P$; **PU**: $\Diamond(\forall S)P$; **NP**: $\Box(\exists S)P$; **PP**: $\Diamond(\exists S)P$; **IU**: $\Box(\forall S)\neg P$; **VU**: $\Diamond(\forall S)\neg P$; **IP**: $\Box(\exists S)\neg P$; **VP**: $\Diamond(\exists S)\neg P$; **BU**: $\Box(\forall S)P \vee \Box(\forall S)\neg P$; **CU**: $\Diamond(\forall S)P \wedge \Diamond(\forall S)\neg P$; **BP**: $\Box(\exists S)P \vee \Box(\exists S)\neg P$; **CP**: $\Diamond(\exists S)P \wedge \Diamond(\exists S)\neg P$.

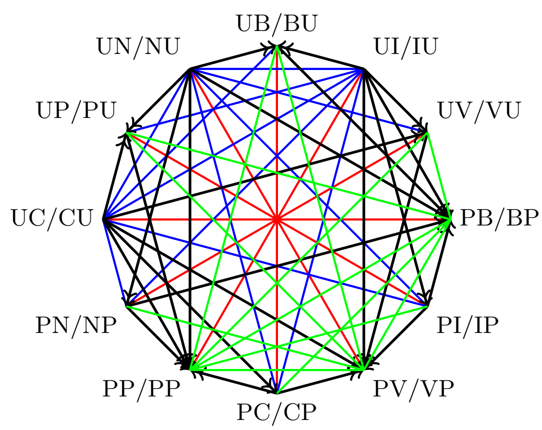

Fig. 1. Schema of a dodecagon of opposition for *de re* and *de dicto* modalities.

Dodecagons. The geometry of *de re* and *de dicto* modalities adheres to the schema in Fig. 1. Red lines connect contradictories (exactly one of which is true), blue lines connect contraries (which cannot be jointly true), green lines connect sub-contraries (which cannot be jointly false) and black arrows connect a modality with its subalterns (logically entailed by the former).

Logical theories. The logical theory of a dodecagon, denoted by $\mathbb{LT}r$ for *de re* modalities and by $\mathbb{LT}d$ for *de dicto* ones, is specified via a list of *inference trees.*

Definition 1 (Inference Tree). *An* inference tree *is a finite set* $T = \{\sigma_1, ..., \sigma_n\}$ *s.t. for* $1 \leq i \leq n$, σ_i *is a finite sequence of sets of wffs (a* branch*); all branches start with the same set of wffs, which is said to be* the root *of* T*. Each set of wffs in a branch is ranked with a progressive number, starting with* **0**.

The relation of immediate inference within a branch σ_i in a tree T is represented by $\rightsquigarrow$. If σ_i is not the sole branch in T, then we use an indexed arrow $\rightsquigarrow_i$.

Definition 2 (Set Derivability - Trees). *A set Γ is derivable from a set Δ in a tree T iff for every branch σ in T, both $\Delta, \Gamma \in \sigma$ and Δ precedes Γ in σ.*

Definition 3 (Set Derivability - Logical Theories). *A set Γ can be derived from a set Δ in a logical theory $\mathbb{LT}$ iff there are trees $T_1, ..., T_{n-1}$ and sets $\Delta_1, ..., \Delta_n$ s.t.: (i) $\Delta = \Delta_1$ and $\Gamma = \Delta_n$, and (ii) for $1 \leq j < n$, Δ_{j+1} can be derived from Δ_j within tree T_j.*

Given two *finite* sets of wffs Γ and Δ, the problem of checking whether Γ can be derived from Δ within a logical theory $\mathbb{LT}$ is the *derivability problem for finite sets* in $\mathbb{LT}$. Below are examples of inference trees shared by $\mathbb{LT}r$ and $\mathbb{LT}d$ (T1), peculiar to $\mathbb{LT}r$ (T2) and peculiar to $\mathbb{LT}d$ (T3):

T1 $\mathbf{0} : \Gamma_0 = \Gamma \cup \{\mathsf{m}_1(\Pi S)\mathsf{m}_2 P\} \rightsquigarrow \mathbf{1} : \Gamma_1 = \Gamma_0 \cup \{\mathsf{m}'_1(\Pi S)\mathsf{m}'_2 P\}$, provided that m_1 and m'_1, as well as m_2 and m'_2, are identical or analogous modalities.
T2 $\mathbf{0} : \Gamma_0 = \Gamma \cup \{\forall S \Box P \vee \forall S \Box \neg P\} \rightsquigarrow_a \mathbf{1} : \Gamma_{1a} = \Gamma_0 \cup \{\forall S \Box P\} \rightsquigarrow_a \mathbf{2} : \Gamma_{2a} = \Gamma_{1a} \cup \{\exists S \Box P, \forall S \Diamond P\} \rightsquigarrow_a \mathbf{3} : \Gamma_{3a} = \Gamma_{2a} \cup \{\exists S \Diamond P\}$.
$\mathbf{0} : \Gamma_0 = \Gamma \cup \{\forall S \Box P \vee \forall S \Box \neg P\} \rightsquigarrow_b \mathbf{1} : \Gamma_{1b} = \Gamma_0 \cup \{\forall S \Box \neg P\} \rightsquigarrow_b \mathbf{2} : \Gamma_{2b} = \Gamma_{1b} \cup \{\exists S \Box \neg P, \forall S \Diamond \neg P\} \rightsquigarrow_b \mathbf{3} : \Gamma_{3b} = \Gamma_{2b} \cup \{\exists S \Diamond \neg P\}$;
T3 $\mathbf{0} : \Gamma_0 = \Gamma \cup \{\Diamond \forall SP \wedge \Diamond \forall S \neg P\} \rightsquigarrow \mathbf{1} : \Gamma_1 = \Gamma_0 \cup \{\Diamond \forall SP, \Diamond \forall S \neg P\} \rightsquigarrow \mathbf{2} : \Gamma_2 = \Gamma_1 \cup \{\Diamond \exists SP, \Diamond \exists S \neg P\}$.

Theorem 1 (Decidability). *The derivability problem for finite sets in $\mathbb{LT}r$ and $\mathbb{LT}d$ is decidable.*

Definition 4 ($\mathcal{L}$-Model). *An $\mathcal{L}$-model [3] is a tuple $\mathfrak{M} = \langle W, R, D, V \rangle$ s.t.:*
1. *W is a non-empty set (called* set of possible worlds*);*
2. *$R \subseteq W \times W$ (called* accessibility relation*);*
3. *D is a non-empty set (called* domain of possible objects*);*
4. *$V : (\mathtt{Pred} \times W) \longrightarrow \wp(D) \setminus \emptyset$ is a* valuation function.

Definition 5 (Truth). *We define $\mathfrak{M}, w \models \phi$ as follows (sample cases):*
1. *$\mathfrak{M}, w \models (\forall S)P$ iff. for all $a \in V(S, w)$, $a \in V(P, w)$.*
2. *$\mathfrak{M}, w \models (\exists S)P$ iff. for some $a \in V(S, w)$, $a \in V(P, w)$.*
3. *$\mathfrak{M}, w \models \Box \psi$ iff. for all $u \in W$ s.t. wRu, $\mathfrak{M}, u \models \psi$.*
4. *$\mathfrak{M}, w \models \Diamond \psi$ iff. for some $u \in W$ s.t. wRu, $\mathfrak{M}, u \models \psi$.*

Definition 6 (Semantic Derivability). *Let Δ and Γ be sets of wffs and C_m the class of all $\mathcal{L}$-models. $\Delta \models_{C_m} \Gamma$ iff. for every $\mathfrak{M} \in C_m$, if $\mathfrak{M} \models \phi$ for all $\phi \in \Delta$, then $\mathfrak{M} \models \psi$ for all $\psi \in \Gamma$.*

Theorem 2 (Soundness). *Syntactic derivations in $\mathbb{LT}r$ and $\mathbb{LT}d$ can be mapped to semantic derivations in C_m.*

References

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